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MMSE and ZF Analysis of Macrodiversity MIMO Systems and Wimax Networks over Flat Rayleigh Fading Paths

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Abstract:
We consider the large scale MIMO systems in which the number of users are gradually increased at that time the receiving antennas performance also decreased gradually. In contrast, almost no analytical results are available for macro diversity systems where both the sources and receive antennas are widely separated. Here, receive antennas experience unequal average SNRs from a source and receiver antenna receives a different average SNR from each source. Although this is an extremely difficult problem,In this paper, we provide approximate distributions for the output SNR of a ZF receiver and the output signal to interference plus noise ratio (SINR) of an MMSE receiver. In addition, simple high SNR approximations are provided for the symbol error rate (SER) of both receivers assuming M-PSK or M-QAM modulations .For better performance of receivers we can also implement the MMSE and ZF analysis in Wimax networks.
Keywords: Macrodiversity, MMSE, ZF, outage probability, optimum combining, zero-forcing, Network MIMO, CoMP

I. INTRODUCTION

Due to the increased demand of wireless communication systems because of the features of the system which provides a wide coverage, high throughput and reliable services, the MIMO systems communication has come into existence. Features provided by these systems ensure the improved system coverage and increased data transmission rate by considering multiple numbers of transmitter and receiver antennas. MIMO systems have fulfilled the necessity of wide coverage, high throughput and reliability of services.

Because of the features of MIMO systems, it became an important part of modern wireless communication [5]. Communication in wireless channels is impaired predominantly by multipath fading. Multipath is the arrival of the transmitted signal at the receiver through differing angles and/or differing time delays and/or differing frequency [4]. MIMO offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency and link reliability and or diversity. Over the last decade, space diversity has further increase the efficiency of communication systems by decoupling the users over channel aware signal processing techniques [3-5]. Also the adaptive equalization techniques have compensated the time dispersion in the channel and thereby increasing the efficiency of data transmission. Many researchers have inclined towards the various processing techniques over the last few years [6,7]. But due to the simplicity, linear equalization techniques have attracted a lot, as they are not optimal in a maximum likelihood sense. Two key equalization techniques having superior features over other equalization techniques are Zero forcing (ZF) and Minimum Mean Square Error (MMSE). Even though these techniques are not optimal, but the MMSE receiver minimizes the mean squared error (MSE) and ZF eliminates the interference completely [8-10]. So far, a rich literature is available over the performance of ZF and MMSE for micro diversity systems, where there is a communication between co-located diversity antenna at the base station and the distributed users [10-12]. But not much research has been done on macro diversity systems whereboth transmit and receive antennas are widely separated.

The reason for the lack of research over macro diversity systems is the complexity of its channel matrix. In micro diversity, Wishart form is used in the classical models and Kronecker correlation matrix. However in macro diversity case, Wishart assumption is not followed, which makes its analytical work extremely difficult. Therefore only few results are available in macrodiversity case [13,14]. In this paper, ZF and MMSE equalization techniques are implemented over macrodiversity case and their performances are compared for flat Rayleigh Frequency selective fading channel for different modulation schemes. BPSK, QPSK and QAM are simulated and compared for the above mentioned scenario.

II. PROPOSED METHOD

In this section, we present the generic system model which is considered throughout this paper. The multiuser MIMO system investigated in this paper consists of *N* distributed single antenna users communicating with *nR*distributed receive antennas in an independent flat Rayleigh fading environment. The *CnR*×1 receive vector is given by

$$\boldsymbol{r} \;=\; \boldsymbol{H}\boldsymbol{s}+\boldsymbol{n}, \qquad \textbf{(1)}$$

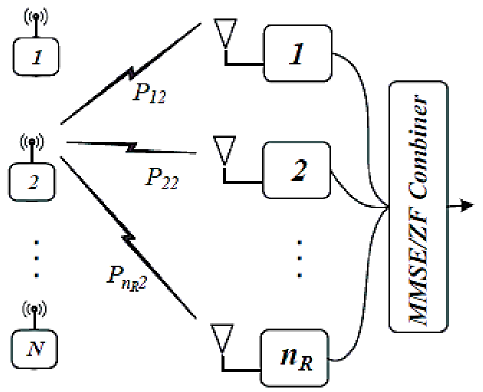


Fig. 2. System diagram. To reduce clutter, only paths from a single source are shown.

where the *CN*×1 data vector, *s* = (*s*1, *s*2, . . . , *sN*)*T* , contains the transmitted symbols from the *N* users and it is normalized, so that *E*_|*si*|2_= 1 for *i* = 1, 2, . . .,*N*. *n* is the *CnR*×1 additive-white-Gaussian-noise (AWGN) vector, *n* ∼ *CN*_0, *σ*2*I* , which has independent entries with *E*_|*ni*|2_= *σ*2, for *i* = 1, 2, . . ., *nR*. The channel matrixcontains independent elements, *Hik*∼*CN*(0, *Pik*), where *E* _|*Hik*|2_= *Pik*. A typical macrodiversity MU-MIMOmultiple access channel (MAC) is shown in Fig. 2, where it isclear that the geographical spread of users and antennas creates a channel matrix *H*, which has independent entries withdifferent *Pik*values.We define the *CnR*×*N*matrix, *P = {Pik},*which holds the average link powers due to shadowing, pathfading, etc.

By assuming that perfect channel state information is available at the receiver side, we consider a system where channel adaptive linear combining is performed at the receiver to suppress multiple access interference [1]. Therefore, the *CN*×1 combiner output vector is ˜*r* = *V Hr*, where *V* is an*CnR*×*N* weight matrix. In this work, And *H* = (*h*1,*h*2, . . . ,hN). Defining *v*2, . . . ,*vN*similarly gives *V* = (*v*1,*v*2, . . . ,*vN*). The vectors, *hk*, clearly play an important role in MMSE combining and it is useful to define the covariance matrix of *hk*by *Pk*= *EhkhHk*_=diag(*P*1*k, P*2*k*, . . . , *PnRk*). From [4], [7], the combining matrix,*V* , and output SNR of the ZF receiver for *nR*≥ *N* aregiven by

$$\boldsymbol{V} = \boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1} \qquad \textbf{(2)}$$

$$\mathrm{SINR} = \boldsymbol{h}_1^H\boldsymbol{R}^{-1}\boldsymbol{h}_1, \qquad (3)$$

Where

$$\boldsymbol{R} = \sum_{k\neq i}^{N}\boldsymbol{h}_k\boldsymbol{h}_k^H + \sigma^2\boldsymbol{I}, \qquad (4)$$

and*H* = (*h*1,*h*2, . . . ,*hN*). Defining *v*2, . . . ,*vN*similarly gives *V* = (*v*1,*v*2, . . . ,*vN*). The vectors, *hk*, clearly play an important role in MMSE combining and it is useful to define the covariance matrix of *hk*by *Pk*= *EhkhHk*_= diag(*P*1*k*, *P*2*k*, . . . , *PnRk*). From [4], [7], the combining matrix, *V* , and output SNR of the ZF receiver for *nR*≥ *N* are given by

$$\boldsymbol{V} = \boldsymbol{H}\left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1} \qquad (5)$$

and

$$\mathrm{SNR} = \frac{1}{\sigma^2\left[\left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}\right]_{11}}. \qquad (6)$$

where[*B*]11 indicates the (1, 1)*th*element of matrix *B*.

III. ZF ANALYSIS

In this section, we derive an approximate CDF for the output SNR of a ZF receiver, a high SNR approximation to SER and also consider some special cases. The following PDFs for the columns of the channel matrix are used throughout the analysis.

3.1. CDF Approximations:

The output SNR of a ZF receiver in (8) can be written as

$$\begin{aligned}\tilde{Z} &= \frac{1}{\sigma^2}\boldsymbol{h}_1^H\left(\boldsymbol{I}-\boldsymbol{H}_2\left(\boldsymbol{H}_2^H\boldsymbol{H}_2\right)^{-1}\boldsymbol{H}_2^H\right)\boldsymbol{h}_1\\ &= \frac{1}{\sigma^2}\boldsymbol{h}_1^H\boldsymbol{M}\boldsymbol{h}_1,\end{aligned} \qquad (7)$$

3.2. High SNR Approximations:

The CF in (24) is a ratio of determinants, where *D* = *I* − 1 *σ*2 *jtP*1. As the SNR grows, *σ*2 → 0 and keeping only thedominant power of *σ*2 in (24) gives

$$\phi_{\tilde{Z}}(t) = \tilde{K}_0\left(\frac{\sigma^2}{-jt}\right)^{n_R-N+1} \qquad (8)$$

Note that when *N* = 2, approximate ˜*K*0 has simpler expression [14], which gives

$$\tilde{K}_0 \simeq \frac{\mathrm{Tr}\left(\boldsymbol{P}_2\right)}{|\boldsymbol{P}_1|\,\mathrm{Tr}\left(\boldsymbol{P}_1^{-1}\boldsymbol{P}_2\right)}. \tag{10}$$

The high SNR SER approximation in (47) has the useful property that all the dependence on P is encapsulated in the $K\tilde{}0$ metric in (50). Hence, $K\tilde{}0$ acts as a stand-alone performance metric as shown in the numerical example in Sec. VII. This feature has implications for systems where only long-term CSI is available for scheduling. Here, $K\tilde{}0$ can be used as a scheduling metric [32] as it is a one-to-one function of the approximate SER. Such situations include systems with rapidly changing channels, systems where CSI exchange is too expensive and systems with large numbers of sources and/or receivers. In all these cases, long term CSI based scheduling may be preferable due to the overheads, delays and errors implicit in obtaining instantaneous CSI

IV. MMSE ANALYSIS

4.1. CDF Approximations:

In this section, we derive the approximate CDF of the output SINR of an MMSE receiver and a high SNR approximation to the SER. Let Z be the output SINR of an MMSE receiver given by (5). Following the same procedure as in the ZF analysis, the CF of Z is

$$\phi_Z(t) \;=\; E\left\{e^{jtZ}\right\} = E\left\{e^{jt\boldsymbol{h}_1^H\boldsymbol{R}^{-1}\boldsymbol{h}_1}\right\} \tag{11}$$

As in the ZF analysis, the PDF and CDF of *Z* can be computed using the identity in [25, eq. 7, 3.382]. Finally we get the approximate PDF of *Z* as

$$\hat{f}_Z(z) = \frac{\Theta\left(\boldsymbol{Q}_2\right)}{\varphi_{n_R}} \sum_{i=1}^{n_R} \eta_i e^{-\omega_i z}, \tag{12}$$

and the CDF of *Z* becomes

$$\hat{F}_Z(z) = \frac{\Theta\left(\boldsymbol{Q}_2\right)}{\varphi_{n_R}} \sum_{i=1}^{n_R} \frac{\eta_i}{\omega_i}\left(1 - e^{-\omega_i z}\right) \tag{13}$$

In contrast to (37), where the ZF SNR is a generalized mixture of *L* exponentials, (68) can be identified as a generalized mixture of $nR \geq L$ exponentials. Since the MMSE SINR has more mixing parameters (*nR*rather than *L*) it might be expected that these increased degrees of freedom will result in a better approximation. Alternatively, the more concise ZF result, which provides a lower bound on the MMSE performance, can be used to provide a simpler expression for use in system design and understanding,

V. SIMULATIONS AND NUMERICAL RESULTS

In this section, we simulate the macrodiversity system shown in Fig. 3, where three base stations (BSs) collaboratevia a central backhaul processing (BPU) in the shaded threesector cluster. This simulation environment was also used in [14] and is sometimes referred to as an edge-excited cell. We consider the three BS scenario having either a single antenna or two antennas each to give *nR*= 3 or *nR*= 6 respectively. In the shaded coverage area of this edge-excited cell, we drop three or four users uniformly in space giving*N*=3

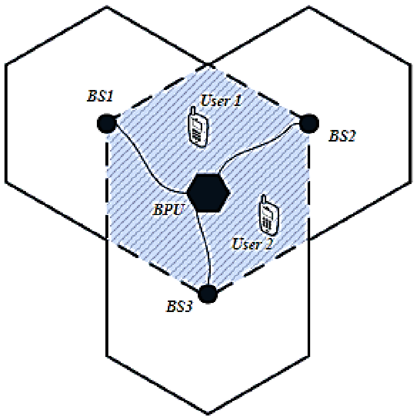
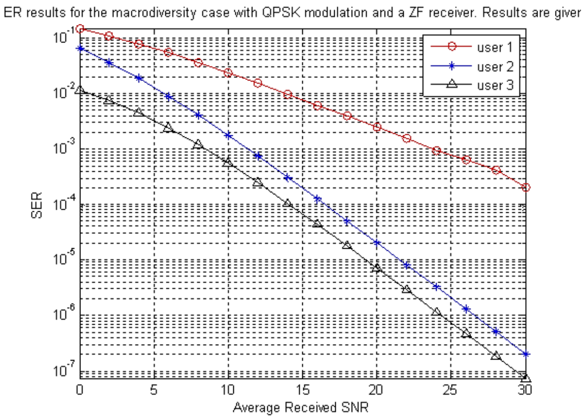


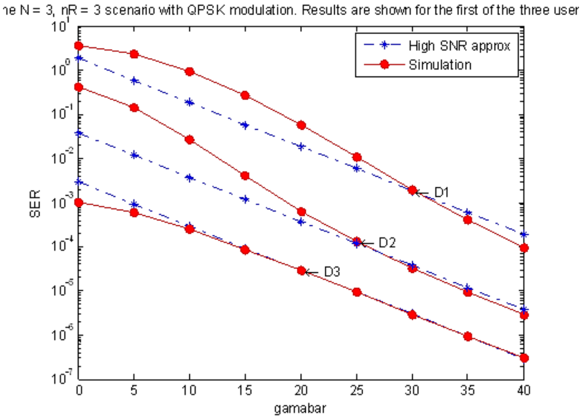
Fig. 3. Network MIMO/edge-excited cell scenario where three base stations serve users in a three-sector cluster. To reduce clutter, only two users are shown.

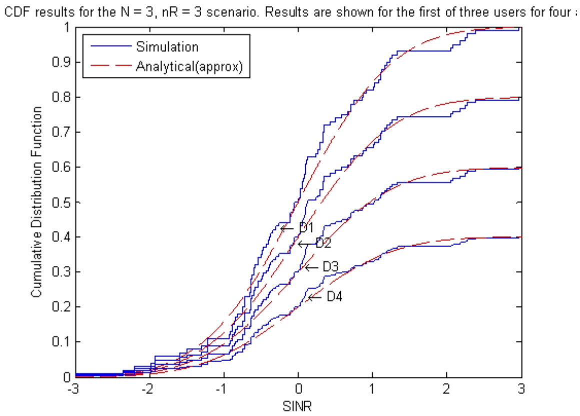
N = 4. For each user, lognormal shadow fading and path loss is considered, where the standard deviation of the



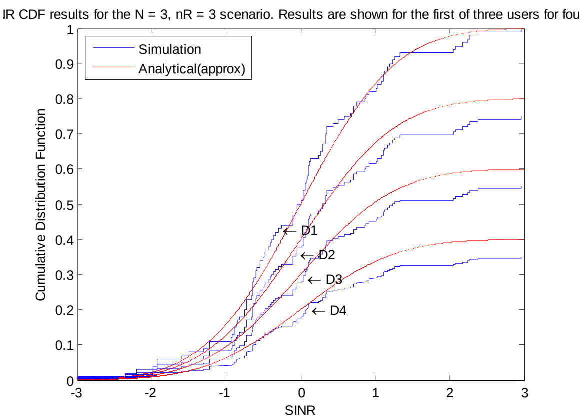
shadowing is 8dB and the path loss exponent is γ = 3.5. The transmit power of the sources is scaled so that the best signal received at the three BS locations is greater than 3dB at least 95% of the time. Even though the analysis in this paper is valid for any set of channel powers, the above methodology allows us to investigate the accuracy of the performance matrices forrealistic sets of channel powers.In Figs. 4, 5 and 6, the case of three single antenna usersand three distributed BSs with a single receiver antenna isconsidered. Here, we investigate both the approximate SINRdistributions and the approximate

SER results for an MMSEreceiver. In Fig. 4, the approximate CDFs of the output SINRare plotted alongside the simulated CDFs. Results are shownfor four random drops and, the results are for a particular user(the first of the three). The agreement between the CDFs isshown to be excellent.

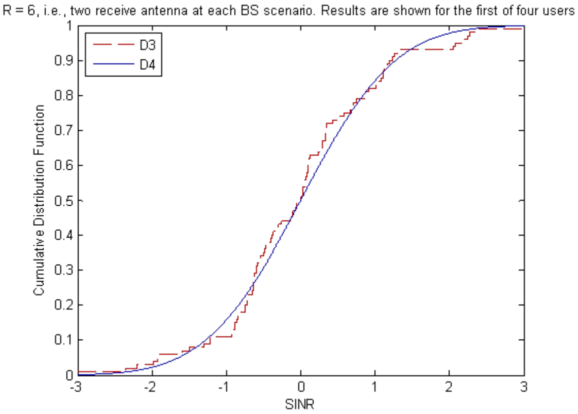




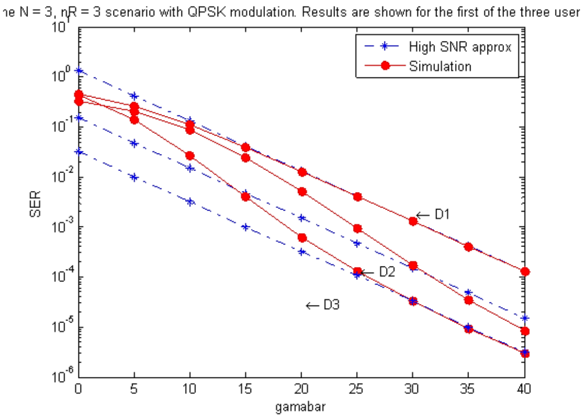
The agreement between the SER results is shown to be excellent across all three drops at SERs below 10−2. Again, this agreement is observed over a wide range with



D1 having much higher SERs than D3. In Fig. 5 and also in Figs. 7-8 the SER is plotted against the transmit SNR, $\bar{\gamma}$. This is chosen instead of the receive SNR to separate the curves so that the drops are visible and are not all

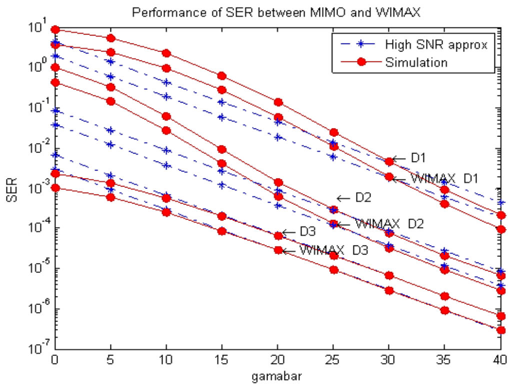


superimposed, which tends to happen when SER is plotted against receive SNR. In Fig. 6, the approximate CDFs of the SNR are plotted alongside the simulated CDFs for a ZF receiver. Results are shown for four random drops.



This is the companion plot to Fig. 4 with the same system but a ZF receiver rather than an MMSE receiver. The accuracy of the results in Fig. 4 and Fig. 6 is interesting, especially when you observe that the Fig. 4 analysis uses (69), a simple mixture of 3 exponentials, and Fig. 6 uses (38) which is a single exponential in this case. In Fig. 7 and 8, the case of four single antenna users and six distributed receive antennas (two at each BS location) is considered. High SNR SER curves are plotted alongside the simulated values. Results are shown for both MMSE (Fig. 7) and ZF (Fig. 8) with QPSK modulation. The agreement between the simulated SER and the high SNR approximation is shown to be less accurate than in Fig. 5, with very close agreement requiring low error rates around 10−4.

The results in Fig. 8 are very informative concerning macrodiversity combining and highlight the difficulties in predicting performance from the *P* matrix.



Consider the simple SIR metric given by the sum of the first column of *P* (the total long term received power from the desired user 1) divided by the sum of columns 2,3 and 4 (the total long term interfering power). In drops D1, D2 and D3 the SIR is -19dB, -2.5dB and 6.5dB. As the SIR increases, the SER in Fig. 8 drops. This is also shown by the $\tilde{K}0$ metric in (50) which gives 17000, 323 and 13 for drops D1, D2 and D3. As SER increases with $\tilde{K}$ 0 both the $\tilde{K}0$ metric and the simple SIR metric give them same performance ranking with D3 the best and D1 the worst. The fourth drop, D4, is the interesting case. Here, the SIR is-10dB, which is lower than both D2 and D3. Hence, from Fig.8 D4 has a better SER performance than D2 and D3 despite having a worse SIR. In order to understand this, consider the *P* matrix for drop D4,

$$\boldsymbol{P}_{D4} = \begin{pmatrix} 0.2061 & 1.3941 & 1.1034 & 4.6938 \\ 0.2061 & 1.3941 & 1.1034 & 4.6938 \\ 2.2923 & 16.8146 & 0.0857 & 0.6790 \\ 2.2923 & 16.8146 & 0.0857 & 0.6790 \\ 0.8361 & 3.4834 & 2.8181 & 0.6700 \\ 0.8361 & 3.4834 & 2.8181 & 0.6700 \end{pmatrix} \quad (14)$$

VI. WIMAX NETWORK:

Worldwide Interoperability for Microwave Access (WiMAX), is a wireless communications technology aiming to provide wireless data over long distances in a variety of ways as an alternative to cable and DSL, from point-to-point links to full mobile cellular type access. It is based on the IEEE 802.16 standard. The name WiMAX was created by the WiMAX Forum, which was formed in June 2001 as an industry-led, not-for-profit organization to promote conformance and interoperability of the standard. The goal of this deliverable is to provide an overview of the functionality and a description of the WiMAX network architecture. We study and assess the coexistence and interoperability solutions between WiMAX and other wireless access networks, such as WLAN (IEEE 802.11) in Beyond 3G (B3G) networks. We also evaluate the special features of the WiMAX technology, such as the improved coverage in Non Line Of Sight (NLOS) environments, in order to examine the applicability of wellknown localization techniques. Finally, we investigate the possibility of developing a new localization technique that exploits the characteristics of WiMAX technology and the underlying network infrastructure to deliver improved positioning accuracy.

VII. CONCLUSION

The performance of MMSE and ZF receivers is well-known in macro diversity systems where the receive antennas are colocated. However, in the macro diversity case, closed form performance analysis is a long-standing, unsolved research problem. In this paper, we make the progress towards solving this problem for the general case of an arbitrary number of transmit and receive antennas. The analysis is based on a derivation which targets the characteristic function of the output SINR. This leads to an expected value which is highly complex in its exact form, but can be simplified by the use of an extended Laplace type approximation. The SINR distribution is shown to have a remarkably simple form as a generalized mixture of exponentials. Also, the asymptotic SER results produce a remarkably compact metric which captures a large part of the functional relationship between the macro diversity power profile and SER.

REFERENCES

[1] S. Verdu, *Multiuser Detection*, 1st edition. Cambridge University Press, 1998.

[2] J. H. Winters, "*Optimum combining in digital mobile radio with cochannel interference*," *IEEE J. Sel. Areas Communications.*, vol. SAC-2, pp. 528–539, July 1984.

[3] H. Gao, P. J. Smith, and M. V. Clark, "*Theoretical reliability of MMSE linear diversity combining in Rayleigh-fading additive interference channels*," *IEEE Trans. Commun.*, vol. 46, no. 5, pp. 666–672, May 1998.

[4] J. H. Winters, J. Salz, and R. D. Gitlin, "*The impact of antenna diversity on the capacity of wireless communication systems*," *IEEETrans. Commun.*, vol. 42, no. 2/3/4, pp. 1740–1751, Feb./Mar./Apr. 1994.

[5] A. Shah, A. M. Haimovich, M. K. Simon, and M.-S. Alouini, "*Exact biterror probability for optimum combining with a Rayleigh fading Gaussian cochannel interferer*," *IEEE Trans. Commun.*, vol. 48, pp. 908–912, Jun. 2000.

[6] M. Chiani, M. Z. Win, A. Zanella, R. K. Mallik, and J. H. Winters, "*Bounds and approximations for optimum combining of*

signals in the presence of multiple cochannel interferers and thermal noise," *IEEETrans. Commun.*, vol. 51, pp. 296–307, Feb. 2003.

[7] D. A. Gore, R. W. Heath, Jr., and A. J. Paulraj, "*Transmit selection in spatial multiplexing systems*," *IEEE Trans. Commun.*, vol. 6, no. 11, pp. 491–493, Nov. 2002.

[8] M. K. Karakayali, G. J. Foschini, and R. A. Valenzuela, "*Network coordination for spectrally efficient communications in cellular systems," IEEE Wireless Commun. Mag.*, vol. 13, no. 4, pp. 56–61, Aug. 2006.

[9] S. Venkatesan, A. Lozano, and R. Valenzuela, "*Network MIMO: overcoming intercell interference in indoor wireless systems,*" in *Proc. 2007IEEE ACSSC*, pp. 83–87.

[10] A. Sanderovich, O. Somekh, H. V. Poor, and S. Shamai "*Uplink macro diversity of limited backhaul cellular network*," *IEEE Trans. Inf. Theory*,vol 55, no. 8, pp. 3457–3478, Aug. 2009.

[11] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communication*, 1st edition. Cambridge University Press, 2007.

[12] M.Sawahashi, Y.Kishiyama, A. Morimoto, D. Nishikawa, and M.Tanno, "*Coordinated multipoint transmission/reception techniques for LTE-Advanced*," *IEEE Wireless Commun. Mag.*, vol. 17, no. 3, pp. 26–34, Aug. 2010.

[13] L. Xiao, L. Dai, H. Zhuang, S. Zhou, and Y. Yao, "*Information-theoretic capacity analysis in MIMO distributed antenna systems*," in *Proc. 2003IEEE VTC*, pp. 779–782.

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